

# Predicting Performance Curves of Centrifugal Pumps In the Absence of OEM Data

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**Abstract:** Chemical and Mechanical Engineers in the oil & gas industry often carry out the task of conducting technical studies to evaluate piping and pipeline systems during events such as pump trips and block valve failures that can lead to pipes cracking at the welded joints, pump impellers rotating in the reverse direction and damaged pipe supports due to excessive vibrations to name a few. Although much literature is available to mitigate such disturbances, a key set of data to conduct transient studies are pump performance curves. A pump performance curve is a plot between pump head and flow rate. In Greenfield projects, when no pump vendor data is available, a necessity arises to use performance curves to conduct pipeline/piping studies to check for parameters such as design pressure.

In Brownfield projects, when a plant undergoes revamp for new process conditions, often existing pumps are reused for different applications with or without impeller trimming. Towards this, pumps need to be re-evaluated for head available for the new process conditions. With wear and tear in pumps systems in ageing facilities that causes deviation from the manufactured OEM pump curves, it becomes difficult to accurately predict if the pump can deliver the required head for the new application.

Traditionally performance curves are provided by the pump original equipment manufacturers (OEM) based on their customized/proprietary models of pump impellers which are designed using methods

such as computational fluid dynamics (CFD) and also field tested to provide guarantee in meeting the requirements of the customer.

The present paper is aimed at applying engineering research in industrial applications for practicing engineers. It provides a methodology called from available literature from past researchers, allowing engineers to predict performance curves for an End Suction single stage radial pump. This article is provided for guidance alone and engineering advice should be sought before application.

**Keywords:** Performance Curves, End Suction Single Stage Radial Type Centrifugal Pump, Euler Turbomachine Equation, Pump Losses

## Introduction

The working principle of a centrifugal pump involves using centrifugal force of a rotating enclosed impeller in a casing to impart energy to a fluid. In doing so, a portion of the energy is lost in the form of mechanical losses with the remaining being transferred to the fluid that raises the fluid's pressure when discharging from the pump casing. A pump impeller consists of vanes that are positioned on a disc to hold fluid and transfer energy as the impeller rotates. The impeller vane geometry is mainly of three types, namely, forward, straight and backward. Backward positioned vanes are popularly used for the reason that with increase in volumetric flow, the power consumed decreases. A representation of the power consumption trends between the three vane geometries is shown in figure 1.

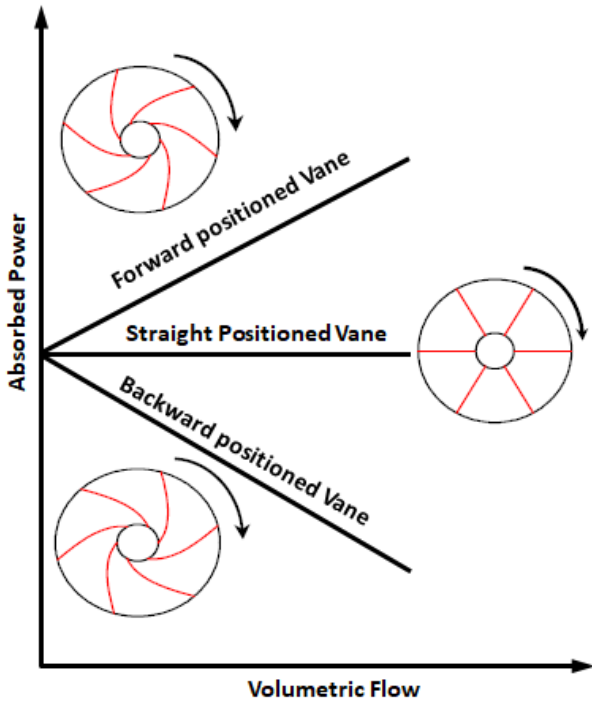


Figure 1. Vane Geometry and Power Consumption

In the current undertaking, a backward vane positioned impeller is chosen considering lower power consumption at higher flow rates for a single stage radial flow pump.

### Principle of Performance Curves

Based on the impeller geometry, performance curves are derived from an aerodynamic analysis of the pump impeller. The basic equation that governs fluid behaviour at the pump's impeller is the Euler's Turbomachine equation relating pump head and fluid velocity. To apply Euler's Equation, the fluid's velocity components are expressed as shown in Fig. 2.

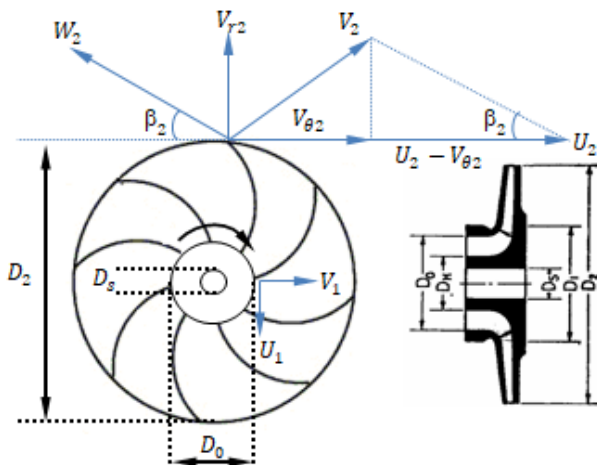


Figure 2. Velocity Triangle of Pump Impeller

### Net Theoretical head Relationship

From the velocity triangle shown in Fig. 1, the net theoretical head is the head developed based on a finite number of vanes in the impeller. The aerodynamic relationship between the net theoretical head ( $H_{Net\ Theoretical}$ ) developed by the fluid for a given impeller speed and its respective velocity components at the impeller inner diameter (ID) represented by subscript '1' and outer diameter (OD) represented by subscript '2' is written as,

$$H_{Theoretical} = \frac{1}{g} [U_2 V_{\theta 2} - U_1 V_{\theta 1}] \quad (1)$$

$$H_{NetTheoretical} = \frac{1}{g} [U_2 V'_{\theta 2} - U_1 V'_{\theta 1}] \quad (2)$$

$$\text{Volumetric Flow, } Q = \pi D_2 b_2 V_{r2} \epsilon_2 \quad (3)$$

From the above relationships, a contraction factor ( $\epsilon_2$ ) is applied to estimate the flow that takes into account the decrease in inlet area of the impeller due to vane thickness. The impeller outlet diameter passage width ( $b_2$ ) is considered to estimate the flow rate ( $Q$ ) into the impeller. The chief parameter based on which other impeller parameters such as vane angle, passage width, number of vanes, etc. are calculated is the impeller inner diameter (ID),  $D_1$  and outer diameter (OD),  $D_2$  for a given impeller speed ( $N$ ).

### Pump Specific Speed ( $N_s$ )

Pump specific speed is a measure to determine what kind of pumps can be selected for a given service. Based on the pump specific speed value, the choice of pumps can vary from radial, Francis Vane, mixed flow or axial flow. The pump specific speed [3] is calculated in metric terms with the below described equation,

$$\text{Specific Speed, } N_s = \frac{N \sqrt{Q}}{H^{3/4}} \left( \frac{\text{rpm} \cdot \frac{m^3}{\text{min}}}{m^4} \right) \quad (4)$$

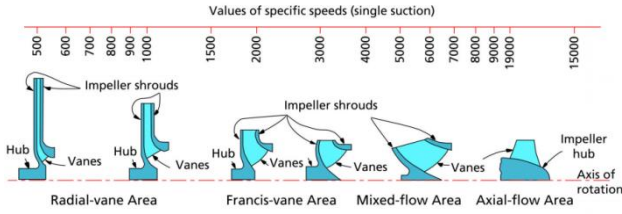


Figure 3. Pump Specific Speed Chart [4]

The above graph shows a distribution of the pump specific speeds based on which the type of pump is selected. Based on the pump speed the volumetric efficiency can be calculated [3] using the relationship,

$$\text{Volumetric Efficiency, } \eta_v = \frac{1}{1 + \frac{1.124}{N_s^{2/3}}} \quad (5)$$

Volumetric efficiency is used to estimate the total flow rate entering into the impeller eye which in turn is used to calculate the impeller eye diameter. Therefore ( $Q_s'$ ) is computed as,

$$\text{Total Flow rate, } Q_s' = \frac{Q}{\eta_v} \quad (6)$$

$Q_s'$  represents the flow that is required to enter the impeller to meet the discharge flow conditions indicated by 'Q' since a portion of the incoming fluid is expected to accumulate in the pump. Hence all impeller design and performance curves calculations are made with  $Q_s'$  to meet conditions of Q.

### Speed and Angular Velocity Relationship

The impeller diameters are calculated by relating the impeller dimensions to the impeller speed (N). The impeller speed is converted to velocity terms, i.e., angular velocity ( $\omega$ ). The relationship between impeller speed and angular velocity is,

$$\text{Angular Velocity, } \omega(m/s) = \frac{2 \times \pi \times N_{rpm}}{60} \quad (7)$$

### Impeller Vane Angle ( $\beta_1, \beta_2$ ) Relationship

It is to be noted that when a fluid is rotated by a surface, a certain amount of slippage occurs between the impeller diameter tip and

the fluid making contact with the impeller tip. This causes the actual fluid velocity leaving the impeller diameter to be slightly lower than the impeller tip speed with the slippage expressed as a 'slip factor' ( $\sigma$ ). The slip factor is incorporated into the velocity triangle relationship to estimate the tangential velocity terms  $V_{\theta 1}$  and  $V_{\theta 2}$ , radial velocity terms  $V_{r1}$  and  $V_{r2}$  as,

$$V_{\theta 1} = U_1 \sigma - \frac{V_{r1}}{\tan \beta_1} \quad (8)$$

$$V_{\theta 2} = U_2 \sigma - \frac{V_{r2}}{\tan \beta_2} \quad (9)$$

The slippage factor ( $\sigma$ ) is computed by relating to the number of vanes (Z) and inlet and outlet diameter vane angle,  $\beta_1$  and  $\beta_2$  as,

$$\sigma = 1 - \frac{\sqrt{\sin \beta_2}}{\beta_1 Z^{0.7}}, \text{ For, } \frac{R_1}{R_2} \leq \varepsilon_{\text{limit}} \quad (10)$$

$$\text{And, } \sigma = 1 - \frac{\sqrt{\sin \beta_2}}{\beta_1 Z^{0.7}} \left[ 1 - \left( \frac{\left( \frac{R_1}{R_2} \right) - \varepsilon_{\text{limit}}}{1 - \varepsilon_{\text{limit}}} \right)^{1/3} \right] \quad (11)$$

For,  $\frac{R_1}{R_2} > \varepsilon_{\text{limit}}$

$$\text{And, } \varepsilon_{\text{limit}} = e^{\left( \frac{-8.16 \sin \beta_2}{Z} \right)} \quad (12)$$

The number of vanes (Z) required is calculated as,

$$Z = 6.5 \times \left( \frac{D_2 + D_1}{D_2 - D_1} \right) \times \sin \left( \frac{\beta_1 + \beta_2}{2} \right) \quad (13)$$

The vane angle at the inner diameter (ID) is computed from the velocity triangle relationship by relating it to the radial component and impeller tip speed as follows,

$$\text{Impeller ID Vane Angle, } \beta_1 = \tan^{-1} \left( \frac{V_{r1}}{U_1} \right) \quad (14)$$

### Impeller Dimensions Relationship

The main parameters required to be estimated are, End of Main Shaft Diameter ( $D_s$ ), Hub Diameter ( $D_H$ ), Hub Length ( $L_H$ ), impeller inlet passage width ( $b_1$ ), impeller outlet passage width ( $b_2$ ) impeller eye diameter ( $D_0$ ), impeller inner diameter ( $D_1$ ), and impeller outlet diameter ( $D_2$ ). The impeller outer diameter ( $D_2$ ) can be calculated using Stepanoff Chart [2]. To calculate the above mentioned parameters, the following equations can be used.

$$\text{Shaft Dia, } D_{sh} = \left[ \frac{P(HP) \times 321000}{N(rpm) \times S_s(psi)} \right]^{1/3} \quad (15)$$

$$\text{Hub Diameter, } D_H = (1.5 \text{ to } 2.0) \times D_{sh} \quad (16)$$

$$\text{Hub Length, } L_H = (1.0 \text{ to } 2.0) \times D_H \quad (17)$$

The fluid velocity at Impeller Eye ( $V_{eye}$ ) is calculated as,

$$V_{eye} = [(0.07 \text{ to } 0.11) + 0.00023N_s] \times \sqrt{2gH} \quad (18)$$

The impeller eye diameter ( $D_0$ ) is taken to be,

$$\text{Impeller Eye Dia, } D_0 = \sqrt{\frac{4 \times Q_s'}{\pi \times V_{eye}}} + D_H^2 \quad (19)$$

$$\text{Impeller OD Tip Speed, } U_2 = K_u \sqrt{2gH} \quad (20)$$

$$\text{OD Radial Velocity, } V_{r2} = K_{m2} \sqrt{2gH} \quad (21)$$

$$\text{ID Radial Velocity, } V_{r1} = K_{m1} \sqrt{2gH} \quad (22)$$

$$\text{Impeller Outer Diameter, } D_2 = \frac{60 \times U_2}{\pi \times N} \quad (23)$$

$$\text{Impeller Outer Diameter, } D_1 = D_2 \left( \frac{D_2}{D_1} \right) \quad (24)$$

$$\text{Impeller ID Tip Speed, } U_1 = \frac{\pi \times D_1 \times N}{60} \quad (25)$$

$$\text{Inlet Passage Width, } b_1 = \frac{Q_s'}{\pi D_1 V_{r1} \epsilon_1} \quad (26)$$

$$\text{Outlet Passage Width, } b_2 = \frac{Q_s'}{\pi D_2 V_{r2} \epsilon_2} \quad (27)$$

The contraction factor ( $\epsilon$ ) for the inner and outer diameters can be estimated by using the thickness of the impeller passage ( $t$ ) at the inlet and outlet diameters as,

$$\text{Contraction factor, } \epsilon = 1 - \frac{Zt}{\pi D \sin \beta} \quad (28)$$

The values of  $K_u$ ,  $K_{m1}$ ,  $K_{m2}$  and  $D_2/D_1$  can be computed from Stepanoff Chart [3],

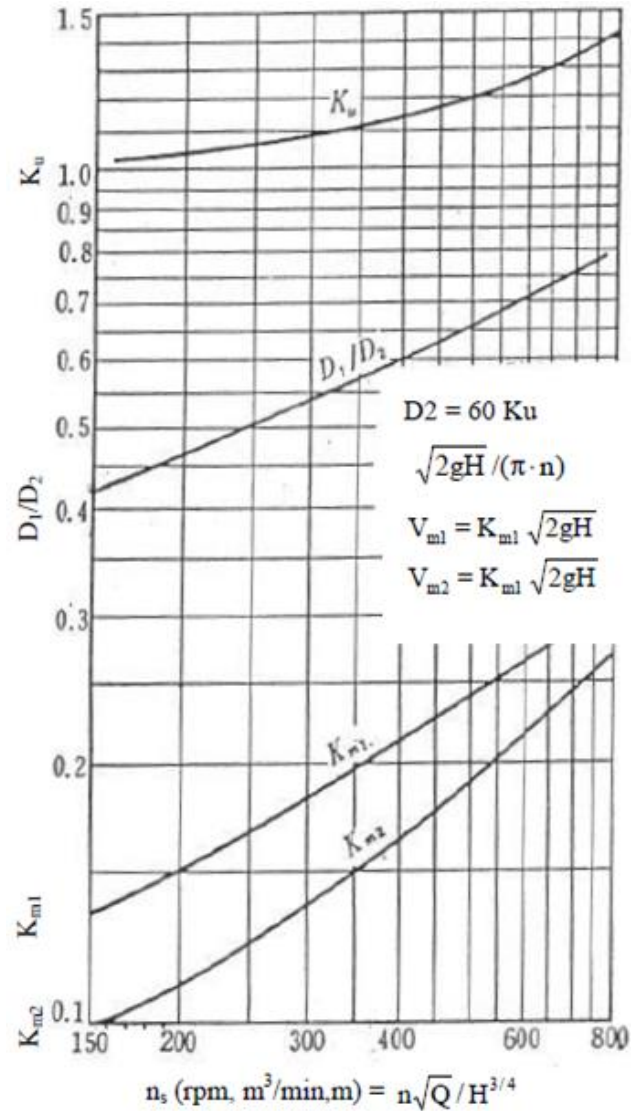


Figure 4. Stepanoff Chart for  $K_u$ ,  $K_{m1}$ ,  $K_{m2}$ ,  $D_2/D_1$

From the equations presented, design procedures can be commenced by assuming ' $\beta_2$ ' and iteratively calculating until the actual head calculated matches with the required pump head. Followed by calculating the net theoretical head, the actual head is calculated by subtracting the pump losses for a range of flow rates.

## Pump Losses

In a realistic scenario, centrifugal pumps experience different forms of mechanical losses. The different types of losses expected during pump operation are (i) Circulation losses, (ii) Inlet Incidence losses, (iii) Surface Friction losses, (iv) Volute Friction losses and (v) Diffusion losses. In addition, parasitic losses are also considered such as (vi) Disc Friction losses and (vii) Recirculation losses. When these losses are subtracted from the theoretical head, the actual head developed by the pump is arrived at. The below figure shows the difference between the net theoretical head and actual pump head.

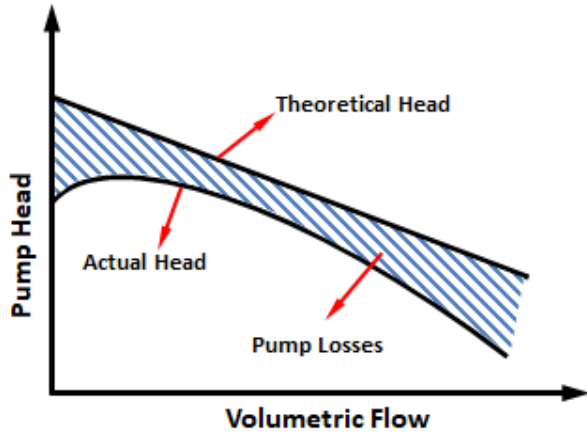


Figure 5. Theoretical Head vs. Actual Head

## Circulation Losses

Circulation flow losses are characterized by circulatory flow that exists within a closed impeller channel when the impeller is rotating. At this point, there would be a mismatch of relative velocity ( $W$ ) between the inlet side and outlet side of the impeller vane. The circulation head is calculated as,

$$H_{circ} = H_{Theoretical} - H_{NetTheoretical} \quad (29)$$

$$H_{circ} = \frac{(U_2 V_{\theta 2} - U_1 V_{\theta 1}) - (U_2 V'_{\theta 2} - U_1 V'_{\theta 1})}{g} \quad (30)$$

$$H_{circ} = \frac{U_2 (V_{\theta 2} - V'_{\theta 2}) + U_1 (V'_{\theta 1} - V_{\theta 1})}{g} \quad (31)$$

$$H_{circ} = \frac{(U_2 \times V_{s2}) + (U_1 \times V_{s1})}{g} \quad (32)$$

The slip velocity is normalized by the impeller tangential velocity as [6],

$$\sigma_s = 1 - \frac{V_s}{U} \quad (33)$$

Therefore the slip velocities at the inlet diameter (ID) and outlet diameter (OD) are,

$$V_{s1} = (1 - \sigma_{s1}) \times U_1 = V'_{\theta 1} - V_{\theta 1} \quad (34)$$

$$V_{s2} = (1 - \sigma_{s2}) \times U_2 = V_{\theta 2} - V'_{\theta 2} \quad (35)$$

With the assumption that the slip factor is nearly equal at both the impeller ID and OD, the whirl velocities are written as,

$$V'_{\theta 1} = U_1 (2 - \sigma_{s1}) - V_{r1} \cot \beta_1 \quad (36)$$

$$V'_{\theta 2} = U_2 \sigma_{s2} - V_{r2} \cot \beta_2 \quad (37)$$

## Inlet Incidence Losses

Incidence flow losses are characterized by losses resulting from a forced change of velocity when fluid enters the pump impeller. When fluid enters the impeller eye in a normal direction, it is followed by a radial change in the direction of fluid flow. Additionally due to difference between the vane inlet angle and angle at which the fluid enters the vane cascade, a loss of head occurs due to forced change in velocity. The incidence losses are calculated as [6],

$$h_{in} = f_{in} \times \frac{(U_1 - V_{\theta 1})^2}{2g} \quad (38)$$

Where,  $f_{in} = 0.5 - 0.7$

$$V_{\theta 1} = U_1 - \frac{Q'_s}{\tan \beta_1 \times \left[ \pi D_1 b_1 - \frac{Z \times t \times b_1}{\sin(\beta_1)} \right]} \quad (39)$$

## Surface Friction Losses

No pump system has perfectly smooth surfaces but instead has some amount of roughness. As a result when the fluid enters the impeller eye, friction is caused between the fluid and the disc surface. Taking into account the losses at the solid boundaries

such as stationary vanes, diffuser and the rest of the impeller surfaces, the surface frictional head loss is calculated as,

$$h_{sf} = \frac{b_2(D_2 - D_1) \times (W_1 + W_2)^2}{2 \times \sin\beta_2 \times H_r \times 4g} \quad (40)$$

$$\text{Where, } H_r = \frac{b_2 \left( \frac{\pi D_2}{Z} \right) \times \sin\beta_2}{b_2 + \left( \frac{\pi D_2}{Z} \right) \times \sin\beta_2} \quad (41)$$

$$\text{Where, } W_1 = \frac{V_{r1}}{\sin\beta_1} \quad (42)$$

$$\text{Where, } W_2 = \frac{V_{r2}}{\sin\beta_2} \quad (43)$$

### Diffusion Losses

Diffusion Losses are characterized by a loss of head when the inlet impeller relative velocity exceeds the outer impeller's relative velocity by a certain factor due to which a portion of the velocity head difference is lost. The diffusion head loss is calculated as [6],

$$h_{DL} = 0.25 \times \left[ \left( \frac{W_1}{W_2} \right)^2 - 2 \right] \frac{W_2^2}{2g} \quad (44)$$

If,  $W_1/W_2 > 1.4$

### Volute Friction Losses

The pump volute receives the fluid pumped by the impeller. Due to its curved shape and changing area, pressure head is lost as the fluid moves towards the discharge flange. Modifying Ref [1] with respect to volute throat area, the volute friction loss is,

$$h_{vf} = 0.8 \times \frac{\left[ V_{\theta 2} \times \left( \frac{D_2}{D_3} \right) \right]^2 - \left[ \frac{Q_s'}{A_3} \right]^2}{2g} \quad (45)$$

$$\text{Assuming that, } D_3 = 1.3 \times D_2 \quad (46)$$

$$\text{Taking Volute Width, } b_3 = 2 \times b_2 \quad (47)$$

$$\text{Volute Throat Area, } A_3 = \pi \times D_3 \times b_3 \quad (48)$$

### Disc Friction Losses

Disc friction losses are the result of a viscous friction between the outside portion of the impeller Disc and the surface of the pump casing. Hence in the case of open impellers, the Disc friction is lower than the case where closed impellers are used. The Disc friction losses can be calculated as [6],

$$P_{df} = C_M \times \rho \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5 \quad (49)$$

Rearranging with  $\rho = Q_s' / v$ ,

$$h_{df} = \frac{P_{df}}{m} = \frac{C_M \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5}{Q_s'} (J/kg) \quad (50)$$

$$h_{df} = \frac{C_M \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5}{Q_s'} \left( \frac{kJ}{kg} \times \frac{1}{1000} \right) \quad (51)$$

$$h_{df} = \frac{C_M \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5}{Q_s'} \left( \frac{102.04}{1000} \right) m \quad (52)$$

$$h_{df} = \frac{0.10204 \times C_M \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5}{Q_s'} m \quad (53)$$

Where,

$$C_M = \left( \frac{k_s}{0.5 \times D_2} \right)^{0.25} \times \left( \frac{s}{0.5 \times D_2} \right)^{0.1} \times \text{Re}^{-0.2} \quad (54)$$

Where,  $b_4$  is the volute width

$$\text{Re} = \frac{U_2 \times D_2 / 2 \times \rho}{\mu} \quad (55)$$

The value of Disc friction loss coefficient ( $C_m$ ) depends on the Disc surface roughness ( $k_s$ ) and also the axial gap width ( $s$ ).

### Recirculation Losses

Recirculation losses are caused due to eddies formed in the pump impeller. The recirculation losses also depend on the size of the impeller in addition to the flow rates into the pump that decide the flow pattern. Hence with larger diameter impellers the

recirculation losses increase. Pumps with high specific speeds also tend to exhibit a higher chance of recirculation. The head loss due to recirculation is estimated as [5],

$$h_{RL} = 0.005 \times \frac{\omega^3 \times D_1^2}{\rho Q} \left(1 - \frac{Q}{Q_0}\right)^{2.5} \quad (56)$$

Where,  $Q_0$  = Design Flow rate

The value of 0.005 for the loss coefficient is described as the default value as per Ref [5]. Using the default value of 0.005, it is observed by the Author to be very high and yields recirculation losses with negative numbers. The recirculation loss coefficient depends on the piping configuration upstream of the pump in addition to the geometrical details of the inlet. The current paper does not account for the upstream piping and the Author iteratively estimates that the recirculation losses coefficient is to be taken in the order of  $1 \times 10^{-3}$  to  $1 \times 10^{-2}$  in order to compensate for the piping losses and arrive at non-negative recirculation loss coefficients.

### Pump Leakage Losses

Pump leakage losses cause a loss of head and subsequently efficiency due to leakages through the Disc and wearing ring. These volumetric losses can be modelled as loss of flow through an orifice. From Ref [8] and Ref [9], the leakage loss can be worked out as,

$$Q_L = C_L \times A_L \times \sqrt{2 \times g \times H_L} \quad (57)$$

From Ref [5], leakage Area is estimated as,

$$A_L = \pi \times D_1 \times b_{cl} \quad (58)$$

And Leakage Head Loss, from Ref [8] as,

$$H_L = \frac{3}{4} \left[ \frac{(U_2^2 - U_1^2)}{2 \times g} \right] \quad (59)$$

Ref [9] provides an approximated value of 0.6 and this has been incorporated into the present undertaking.

As per Ref [8], a wearing ring clearance of 0.01 inch for rings up to 6 inch diameter or less is a good practice. For rings greater than 6 inches and up to 12 inch, the clearance is increased by 0.001 inch for every inch of ring diameter. For over 12 inch, increase by 0.0005 inches per inch of ring diameter over 12 inches. Therefore the clearance width taking into consideration the above criteria,

$$b_{cl} = 0.01in + 0.001 \times \left[ \left( \frac{D_2 + D_1}{2} \right) - 6in \right] \quad (60)$$

### Actual Pump Head

The Actual Pump Head is calculated by subtracting all the different head losses calculated from the theoretical pump head. Therefore the actual head ( $H_{Act}$ ) is,

$$H_{Act} = H_{NT} - (h_{circ} + h_{in} + h_{sf} + h_{dL} + h_{vf} + h_{df} + h_{RL}) \quad (61)$$

### A Case Study

To understand and validate the described methodology, procedures are applied to estimate the performance curves for a certain model of an industrial water pump with a chosen set of process data. The pump model used for validation is a Grundfos Model No. NB 200-400/392, 4 Pole, 50 Hz, End Suction single stage centrifugal pump, Ref [10]. The table below gives a summary of the input data used to predict the performance curves.

**Table 1. Input Process and Mechanical Data**

Service	Industrial Water
Flow Rate [Q]	364 m <sup>3</sup> /h
Rotational Speed [N]	1493 rpm
Operating Temperature	25°C
Fluid Density [ $\rho$ ]	973.6 kg/m <sup>3</sup>
Suction Pressure [ $P_1$ ]	5.0 bara
Discharge Pressure [ $P_2$ ]	10.0 bara
Required Head [H]	52.4 m

OEM Pump Efficiency [ $\eta_p$ ]	73.1 %
Motor Rated Capacity	110 kW
OEM Impeller ID [ $D_2$ ]	392 mm

## Results

With the data presented in Table 1, calculations were performed and repeated as shown in Table 3 for various range of pump flow rates to arrive at the pump performance curves as shown in below (H vs. Q,  $\eta$  vs. Q).

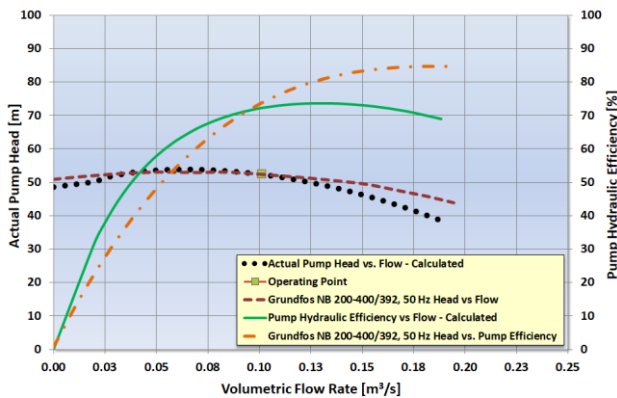


Figure 6. Calculated Pump Performance Curve

In deriving the performance curves, the min/max operable region is assigned for a range of 80% to 110% of the best efficiency point (BEP) while the preferred region of operation is 70% to 120% of BEP to minimize failure due to seal and bearing failure. A plot is made between manufacturer's data and predicted pump performance curves to assess the deviation as shown below.

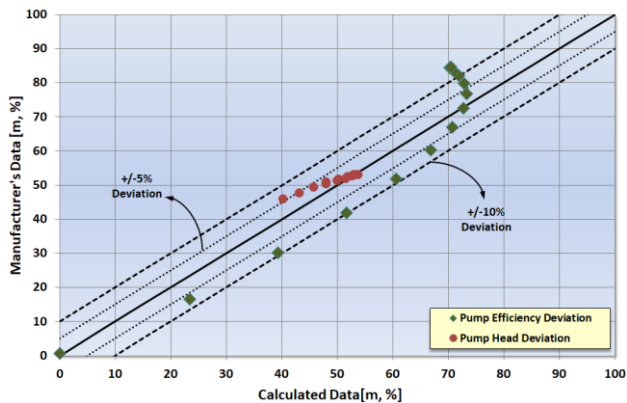


Figure 7. Deviation between Predicted and OEM Values

From the deviation calculated between the predicted pump performance data and

manufacturer's data, the percentage deviation for predicted pump head is largely within  $\pm 5\%$  for most data points. The pump hydraulic efficiency calculated however shows a deviation of most points in the range of  $\pm 10\%$  with respect to manufacturer's data. The pump curve upon approaching shut-off head droops towards y-axis indicating a fall in head as the pump approaches zero flow. This is a characteristic of end suction centrifugal pumps where the volute friction losses begin to increase at lower flow rates contributing to a decrease in pump head as shut-off conditions approach. The key impeller geometry parameters calculated is shown in Table 2 as follows,

Table 2. Calculated Impeller Parameters

Diameter of Main Shaft End	48 mm
Hub Diameter	84 mm
Hub Length	126 mm
Diameter of Impeller Eye	195 mm
Impeller Outer Diameter	392 mm
Impeller Inner Diameter	177 mm
Vane Angle at Inlet	27.6°
Vane Angle at Outlet	19°
Number of Impeller Vanes	7
Blade Thickness	3.175 mm
Inlet Impeller Passage Width	57 mm
Outlet Impeller Passage Width	26 mm

## Affinity Laws

Pump flow can be treated as incompressible flow since liquids are largely incompressible. Fan laws can be used to derive performance curves for various speeds based on the following relationships.

$$Q \propto N \quad (62)$$

$$H \propto N^2 \quad (63)$$

Constants 'k<sub>1</sub>' and 'k<sub>2</sub>' can be estimated for the base speed of 1493 rpm by re-writing as,

$$Q = k_1 N \Rightarrow k_1 = \left( \frac{Q}{N} \right)_{1493 \text{ rpm}} \quad (64)$$

$$H = k_2 N^2 \Rightarrow k_2 = \left( \frac{H}{N^2} \right)_{1493 \text{ rpm}}^2 \quad (65)$$

The calculated values of 'k<sub>1</sub>' and 'k<sub>2</sub>' can be used to estimate the H vs. Q performance curves for speeds of 60%, 70%, 80% and 90%, as shown below.

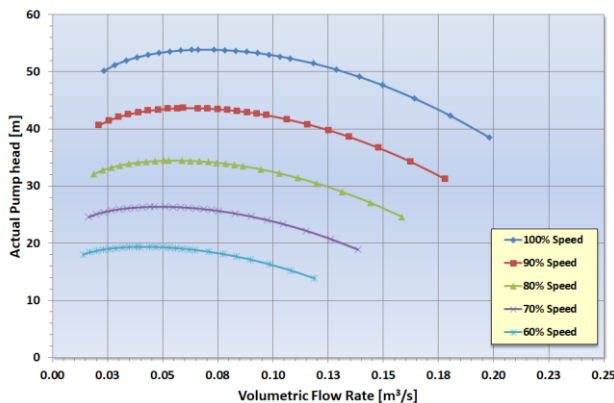


Figure 8. Pump Performance Curves – Various Speeds

### Technical Notes

1. For a given set of hydraulic conditions, a centrifugal pump is designed to operate for one set of flow and head. Deviation from this operating point is allowed only to some degree.
2. Pump selection closer to the BEP will yield a more efficient pump with the least amount of vibration and radial forces acting on the shaft. Pump system resistance curve should be calculated accurately because the pump operates where the performance curve intersects the system curve.
3. In the case of single volute pumps, operating away from the BEP will cause the shaft to deflect with bearings and seals rubbing against the casing components. The fluid flow angle into the impeller will also not align to match impeller speeds and vane angles causing suction recirculation, fluid to stall and cavitation.
4. It is not always possible to operate the pump at the BEP for the conditions required and hence a flow variation of  $\pm 10\%$  of BEP is allowed.
5. Minimum stable continuous flow (MSCF) is the minimum flow below which the pump is not allowed to operate. Although API 610 recommends that the rated region is located between 80% and 110% of BEP the preferred region of flow is between 70% and 120% of BEP.
6. Clause 6.1.12 of API 610 11th edition states *“Setting limits for preferred operating region and the location of rated flow is not intended to lead to the development of additional sizes of small pumps or preclude the use of high-specific-speed pumps. Small pumps that are known to operate satisfactorily at flows outside of the specified limits and high specific speed pumps that may have a narrower preferred operating region than specified should be offered...”* Therefore the Allowable Operating Region is set by the manufacturer as the allowable region to operate with stability whilst conforming to predefined API 610 vibration limits.
7. The Net Positive Suction Head Available (NPSH<sub>A</sub>) should always be higher than the (NPSH<sub>R</sub>) required.
8. Pumps that are expected to operate less frequent can be chosen such that they operate at lower speeds at the cost of efficiency. Since the pump is selected to operate intermittently, a slightly lower efficiency pump is acceptable compared to a higher speed pump. This will ensure a longer operating life cycle.

## Calculations for Pump Performance Curves

A Set of calculations is presented to demonstrate the methodology for pump data presented.

**Table 3. Pump Performance Curves Calculations**

Output Data	
Shaft Dimensions	
Shaft Power [L]	$L = \frac{m \times H}{\eta_0} = \frac{0.1011 m^3/s \times 973.6 kg/m^3 \left( \frac{52.4}{102.04} \right) kJ/kg}{0.731} = 69.1 kW$
Pump Specific Speed [N <sub>s</sub> ]	$N_s = \frac{N \times \sqrt{Q}}{H^{3/4}} = \frac{1493 \times \sqrt{6.067}}{(52.4)^{3/4}} = 189 \frac{rpm \sqrt{m^3/min}}{m^{3/4}}$
Main Shaft End Diameter [d <sub>sh</sub> ]	$D_{sh} = \left[ \frac{P(HP) \times 321000}{N(rpm) \times S_s(psi)} \right]^{1/3} = \left[ \frac{125 \times 321000}{1493 \times 4000} \right]^{1/3} = 1.89 inch (\sim 48mm)$
Volumetric Efficiency [η <sub>v</sub> ]	$\eta_v = \frac{1}{1 + \frac{1.124}{N_s^{2/3}}} = \frac{1}{1 + \frac{1.124}{189^{2/3}}} = 0.967 \text{ or } 96.7\%$
Hub Dimensions	
Hub Diameter [D <sub>H</sub> ]	$D_H = (1.5 \text{ to } 2.0) \times D_{sh}, \text{ Taking } 1.75, D_H = 1.75 \times 48mm = 84mm$
Hub Length [L <sub>H</sub> ]	$L_H = (1.0 \text{ to } 2.0) \times D_H, \text{ Taking } 1.5, L_H = 1.5 \times 84 = 126mm$
Impeller Dimensions	
Total Flow Rate [Q <sub>s</sub> ']	$Q_s' = \frac{Q}{\eta_v} = \frac{0.1011}{0.967} = 0.1045 m^3/s$
Velocity of Liquid at Impeller Eye [V <sub>eye</sub> ]	$V_{eye} = [(0.07 \text{ to } 0.11) + 0.00023 N_s] \times \sqrt{2gH}, \text{ Taking } 0.09, \\ V_{eye} = [0.09 + (0.00023 \times 190)] \times \sqrt{2 \times 9.81 \times 52.4} = 4.3 m/s$
Diameter of Impeller Eye [D <sub>0</sub> ]	$D_0 = \sqrt{\frac{4 \times Q_s'}{\pi \times V_{eye}}} + D_H^2 = \sqrt{\frac{4 \times 0.1045}{\pi \times 4.3}} + \left( \frac{84}{1000} \right)^2 = 0.195m \text{ or } 195mm$
Coefficients 'K <sub>U</sub> '	1.043 (From Stepanoff Charts, Fig. 4)
'K <sub>m1</sub> '	0.149 (From Stepanoff Charts, Fig. 4)
'K <sub>m2</sub> '	0.113 (From Stepanoff Charts, Fig. 4)
'D <sub>2</sub> /D <sub>1</sub> '	0.452 (From Stepanoff Charts, Fig. 4)

$$\text{Angular Velocity } [\omega] \quad \omega = \frac{2 \times \pi \times N_{rpm}}{60} = \frac{2 \times \pi \times 1493}{60} = 156 \text{ m/s}$$

#### Impeller

**Outer Diameter [D<sub>2</sub>]** Taken as D<sub>2</sub> = 0.392m (392 mm) from Table 1)

$$\text{Impeller OD Tip Speed [U}_2\text{]} \quad U_2 = \frac{\omega D_2}{2} = \frac{156 \times 0.392}{2} = 30.6 \text{ m/s}$$

$$\text{Impeller Inner Diameter [D}_1\text{]} \quad D_1 = D_2 \left( \frac{D_2}{D_1} \right) = 0.392 \times 0.452 = 0.177 \text{ m or } 177 \text{ mm}$$

$$\text{Impeller ID Tip Speed [U}_1\text{]} \quad U_1 = \frac{\pi N D_1}{60} = \frac{\pi \times 1493 \times 0.177}{60} = 13.9 \text{ m/s}$$

#### Inlet Flow

$$\text{Radial Velocity [V}_{r1}\text{]} \quad V_{r1} = K_{m1} \sqrt{2gH} = 0.149 \times \sqrt{2 \times 9.81 \times 52.4} = 4.8 \text{ m/s}$$

#### Outlet Flow

$$\text{Radial Velocity [V}_{r2}\text{]} \quad V_{r2} = K_{m2} \sqrt{2gH} = 0.113 \times \sqrt{2 \times 9.81 \times 52.4} = 3.6 \text{ m/s}$$

### Vane Dimensions

**Vane Angle at Outlet [β<sub>2</sub>]** Assume β<sub>2</sub> = 27.6° (Note: To be solved iteratively till H<sub>Actual</sub> = H<sub>Required</sub>)

$$\text{Vane Angle at Inlet [β}_1\text{]} \quad \beta_1 = \tan^{-1} \left( \frac{V_{r1}}{U_1} \right) = \tan^{-1} \left( \frac{4.8}{13.9} \right) \approx 19^\circ$$

$$\begin{aligned} \text{Number of Impeller Vanes [Z]} \quad Z &= 6.5 \times \left( \frac{D_2 + D_1}{D_2 - D_1} \right) \times \sin \left( \frac{\beta_1 + \beta_2}{2} \right) \\ Z &= 6.5 \times \left( \frac{0.392 + 0.177}{0.392 - 0.177} \right) \times \sin \left( \frac{27.6 + 19}{2} \right) = 6.84 \Rightarrow Z = 7 \end{aligned}$$

$$\varepsilon_{\lim it 2} = e^{\left( \frac{8.16 \sin \beta_2}{Z} \right)} = e^{\left( \frac{8.16 \times \sin(27.6)}{7} \right)} = 0.583$$

#### Check

$$\frac{R_1}{R_2} \leq \varepsilon_{\lim it} \quad \frac{R_1}{R_2} = \frac{0.177}{0.392} = 0.452 < \varepsilon_{\lim it 1} < \varepsilon_{\lim it 2},$$

$$\text{Therefore, } \sigma = 1 - \frac{\sqrt{\sin \beta_2}}{\beta_1 Z^{0.7}}$$

$$\text{Slippage Factor } [\sigma_{s1}] \quad \sigma_{s1} = 1 - \frac{\sqrt{\sin \beta_2}}{\beta_1 Z^{0.7}} = 1 - \frac{\sqrt{\sin(27.6)}}{19 \times 7^{0.7}} = 0.991$$

**Slippage  
Factor [ $\sigma_{s2}$ ]**

$$\sigma_{s2} = 1 - \frac{\sqrt{\sin \beta_2}}{\beta_1 Z^{0.7}} = 1 - \frac{\sqrt{\sin(27.6)}}{19 \times 7^{0.7}} = 0.991$$

**Blade  
Thickness [t]**

Taking, 0.125 inches (0.0032m)

**Thickness of  
Inlet Impeller  
Passage [ $t_1$ ]**

Taking, 0.3175 inches (0.0081m)

**Thickness of  
Outlet  
Impeller  
Passage [ $t_2$ ]**

Taking, 0.3175 inches (0.0081m)

**Inlet  
Contraction  
Factor [ $\varepsilon_1$ ]**

$$\varepsilon_1 = 1 - \frac{Z t_1}{\pi D_1 \sin \beta_1} = 1 - \frac{7 \times 0.0081}{\pi \times 0.177 \times \sin(19)} = 0.688$$

**Outlet  
Contraction  
Factor [ $\varepsilon_2$ ]**

$$\varepsilon_2 = 1 - \frac{Z t_2}{\pi D_2 \sin \beta_2} = 1 - \frac{7 \times 0.0081}{\pi \times 0.392 \times \sin(27.6)} = 0.901$$

**Inlet Impeller  
Passage  
Width [ $b_1$ ]**

$$b_1 = \frac{Q'_s}{\pi D_1 V_{r1} \varepsilon_1} = \frac{0.1045}{\pi \times 0.177 \times 4.8 \times 0.688} \approx 0.057m (57mm)$$

**Outlet  
Impeller  
Passage  
Width [ $b_1$ ]**

$$b_2 = \frac{Q'_s}{\pi D_2 V_{r2} \varepsilon_2} = \frac{0.1045}{\pi \times 0.392 \times 3.6 \times 0.901} = 0.026m (26mm)$$

**Clearance  
Width ( $b_{cl}$ )**

$$b_{cl} = 0.01in + 0.001 \times \left[ \left( \frac{D_2 + D_1}{2} \right) - 6in \right]$$

$$b_{cl} = 0.01 + 0.001 \times \left[ \left( \frac{0.392 + 0.177}{2} \times \frac{1000}{25.4} \right) - 6 \right] = 0.0152in (\sim 0.4mm)$$

**Leakage Area  
( $A_L$ )**

$$A_L = \pi \times D_1 \times b_{cl} = \pi \times 0.177 \times \frac{0.4}{1000} = 0.000223m^2$$

**Leakage Head  
Loss [ $H_L$ ]**

$$H_L = \frac{3}{4} \left[ \frac{(U_2^2 - U_1^2)}{2 \times g} \right] = \frac{3}{4} \times \left[ \frac{(30.6^2 - 13.9^2)}{2 \times 9.81} \right] = 28.5m$$

**Leakage Head  
Loss [ $Q_L$ ]**

$$Q_L = 0.6 \times 0.000223 \times \sqrt{2 \times 9.81 \times 28.5} = 0.00317m^3/s$$

**Outlet  
Tangential  
Velocity [ $V_{\theta 1}$ ]**

$$\text{Adding Leakage Loss, } Q'_s = Q_s' + Q_L, \quad V_{\theta 1} = U_1 - \frac{Q'_s}{\tan \beta_1 \times \left[ \pi D_1 b_1 - \frac{Z \times t \times b_1}{\sin(\beta_1)} \right]}$$

$$V_{\theta 1} = 13.9 - \frac{0.1045 + 0.00317}{\tan(19^\circ) \times \left[ \pi \times 0.177 \times 0.057 - \frac{7 \times 0.0032 \times 0.057}{\sin(19^\circ)} \right]} \approx 2.7 \text{ m/s}$$

**Outlet  
Tangential  
Velocity [V<sub>θ2</sub>]**

$$V_{\theta 2} = U_1 - \frac{Q'_s}{\tan \beta_2 \times \left[ \pi D_2 b_2 - \frac{Z \times t \times b_2}{\sin(\beta_2)} \right]}$$

$$V_{\theta 2} = 30.6 - \frac{0.1045 + 0.00317}{\tan(27.6^\circ) \times \left[ \pi \times 0.392 \times 0.026 - \frac{7 \times 0.0032 \times 0.026}{\sin(27.6^\circ)} \right]} \approx 24 \text{ m/s}$$

**Actual Whirl  
Velocity at  
Inlet [V<sub>θ1</sub>']**

$$V'_{\theta 1} = U_1(2 - \sigma_{s1}) - V_{r1} \cot \beta_1 = [13.9 \times (2 - 0.991)] - \frac{4.8}{\tan(19^\circ)} = 0.09 \text{ m/s}$$

**Actual Whirl  
Velocity at  
Inlet [V<sub>θ2</sub>']**

$$V'_{\theta 2} = U_2 \sigma_{s2} - V_{r2} \cot \beta_2 = (30.6 \times 0.991) - \frac{3.6}{\tan(27.6^\circ)} \approx 23.5 \text{ m/s}$$

**Slip Velocity  
at Inlet [V<sub>s1</sub>]**

$$V_{s1} = (1 - \sigma_{s1}) \times U_1 = V'_{\theta 1} - V_{\theta 1} = 0.09 - 2.7 = -2.6 \text{ m/s}$$

**Slip Velocity  
at Inlet [V<sub>s2</sub>]**

$$V_{s2} = (1 - \sigma_{s2}) \times U_2 = V_{\theta 2} - V'_{\theta 2} = 24 - 23.3 = 0.7 \text{ m/s}$$

**Net  
Theoretical  
Head  
[H<sub>NetTheoretical</sub>]**

$$H_{NetTheoretical} = \frac{1}{g} [U_2 V'_{\theta 2} - U_1 V'_{\theta 1}] = \frac{[(30.6 \times 23.5) - (13.9 \times 0.09)]}{9.81} = 73.1 \text{ m}$$

**Theoretical  
Power  
Absorbed  
[P<sub>Theoretical</sub>]**

$$P_{NetTheoretical} = \frac{Q'_s \times \rho \times H}{102.04} = \frac{(0.1045 + 0.00317) \times 973.6 \times 73.1}{102.04} = 75.1 \text{ kW}$$

**Theoretical  
Shut-off Head  
[H<sub>Shut-off</sub>]**

$$H_{Shut-off} = \frac{U_2^2 - U_1^2}{g} = \frac{30.6^2 - 13.9^2}{9.81} \approx 75.6 \text{ m}$$

## Pump Losses

### Circulation Losses

**Circulation  
Head Loss**

$$H_{Circ} = \frac{(U_2 V_{s2} + U_1 V_{s1})}{g} = \frac{[(30.6 \times 0.7) + (13.9 \times (-2.6))]}{9.81} \approx -1.5 \text{ m}$$

### Inlet Incidence Losses

**Inlet  
Incidence  
Head Loss**

$$h_{in} = f_{in} \times \frac{(U_1 - V_{\theta 1})^2}{2g}, \text{ Taking, } f_{in}=0.5, h_{in} = 0.5 \times \frac{(13.9 - 2.7)^2}{2 \times 9.81} = 3.2 \text{ m}$$

## Surface Friction Losses

**Inlet Relative Velocity [W<sub>1</sub>]**  $W_1 = \frac{V_{r1}}{\sin\beta_1} = \frac{4.8}{\sin(19)} \approx 15m/s$

**Inlet Relative Velocity [W<sub>2</sub>]**  $W_2 = \frac{V_{r2}}{\sin\beta_2} = \frac{3.6}{\sin(27.6)} \approx 8m/s$

**Hydraulic Diameter [D<sub>H</sub>]**  $H_R = \frac{b_2 \left( \frac{\pi D_2}{Z} \right) \times \sin\beta_2}{b_2 + \left( \frac{\pi D_2}{Z} \right) \times \sin\beta_2} = \frac{0.026 \times \left( \frac{\pi \times 0.392}{7} \right) \times \sin(27.6)}{0.026 + \left( \frac{\pi \times 0.392}{7} \right) \times \sin(27.6)} \approx 0.02m$

**Surface Friction Losses**

$$h_{sf} = \frac{b_2 \times (D_2 - D_1) \times (W_1 + W_2)^2}{2 \times \sin\beta_2 \times H_r \times 4g}$$

$$h_{sf} = \frac{0.026 \times (0.392 - 0.177) \times (8 + 15)^2}{2 \times \sin(27.6) \times 0.02 \times 4 \times 9.81} \approx 4.1m$$

## Volute Friction Losses

**Volute Throat Diameter [D<sub>3</sub>]** Assuming,  $D_3 = 1.3 \times D_2 = 1.3 \times 0.392 = 0.51m$

**Volute Width [b<sub>3</sub>]**  $b_3 = 2 \times b_2 = 2 \times 0.026 = 0.052m$

**Volute Throat Area [A<sub>3</sub>]**  $A_3 = \pi \times D_3 \times b_3 = \pi \times 0.5096 \times 0.052 = 0.084m^2$

**Volute Friction Loss Head**

$$h_{vf} = 0.8 \times \frac{\left[ V_{\theta 2} \times \left( \frac{D_2}{D_3} \right) \right]^2 - \left[ \frac{Q_s'}{A_3} \right]^2}{2g}$$

$$h_{vf} = 0.8 \times \frac{\left( 24 \times \left( \frac{0.392}{0.51} \right) \right)^2 - \left( \frac{0.1034 + 0.00317}{0.084} \right)^2}{2 \times 9.81} = 13.8m$$

## Disc Friction Losses

**Disc Friction Loss Head** Taking Disc Surface Roughness [k<sub>s</sub>] = 5 microns (5 × 10<sup>-6</sup>)

Axial Gap [s] = 12.7 mm (1.27 × 10<sup>-2</sup>)

Viscosity of Water at 25°C = 0.00091 kg/m.s

**Reynolds Number**  $Re = \frac{U_2 \times \frac{D_2}{2}}{\mu} = \frac{30.6 \times 0.392/2 \times 973.6}{0.00091} = 6,416,773$

**Disc Coefficient Friction [C<sub>M</sub>]**  $C_M = \left( \frac{k_s}{0.5 \times D_2} \right)^{0.25} \times \left( \frac{s}{0.5 \times D_2} \right)^{0.1} \times Re^{-0.2}$

$$C_M = \left( \frac{5 \times 10^{-6}}{0.5 \times 0.392} \right)^{0.25} \times \left( \frac{0.0127}{0.5 \times 0.392} \right)^{0.1} \times 6416773^{0.2} = 2.35 \times 10^{-3}$$

**Disc Friction  
Loss Head**

$$h_{df} = \frac{0.10204 \times C_M \times \omega^3 \times \left( \frac{D_2 - D_1}{2} \right)^5}{Q_s'}$$

$$h_{df} = \frac{0.10204 \times 2.35 \times 10^{-3} \times 156^3 \times \left( \frac{0.392 - 0.177}{2} \right)^5}{0.1045 + 0.00317} \approx 0.1m$$

### Recirculation Losses

**Recirculation  
Loss Head**

$$h_{RL} = 0.005 \times \frac{\omega^3 \times D_1^2}{\rho Q} \left( 1 - \frac{Q}{Q_0} \right)^{2.5}, \text{ Here the value of 0.005 is replaced with 0.00075}$$

Taking Max Flow Rate,  $Q_0 = 110\%$  of Rated Flow =  $1.1 \times 1.1 = 0.11 \text{ m}^3/\text{s}$

$$h_{RL} = 0.00075 \times \frac{\omega^3 \times D_1^2}{\rho Q_s'} \left( 1 - \frac{Q_s'}{Q_0} \right)^{2.5}$$

$$h_{RL} = 0.00075 \times \frac{156^3 \times 0.177^2}{973.6 \times (0.1045 + 0.00317)} \times \left( 1 - \frac{0.1045 + 0.00317}{0.11} \right)^{2.5} \approx 0m$$

### Diffusion Losses

**Diffusion Loss  
Head**

$$h_{DL} = 0.25 \times \left[ \left( \frac{W_1}{W_2} \right)^2 - 2 \right] \frac{W_2^2}{2g}, \text{ If, } W_1/W_2 > 1.4$$

$$W_1 = \left( \frac{V_{r1}}{\sin \beta_1} \right) = \left[ \frac{4.8}{\sin(19)} \right] \approx 15m/s; W_2 = \left( \frac{V_{r2}}{\sin \beta_2} \right) = \left[ \frac{3.6}{\sin(27.6)} \right] \approx 8m/s$$

$$\frac{W_1}{W_2} = \frac{15}{8} = 1.875 > 1.4$$

$$h_{DL} = 0.25 \times \left[ \left( \frac{W_1}{W_2} \right)^2 - 2 \right] \frac{W_2^2}{2g} = 0.25 \times \left[ \left( \frac{15}{8} \right)^2 - 2 \right] \times \frac{8^2}{2 \times 9.81} = 1.24m$$

**Actual Head  
[H<sub>Actual</sub>]**

$$H_{Actual} = H_{NetTheoretical} - (h_{circ} + h_{in} + h_{sf} + h_{vf} + h_{df} + h_{RL} + h_{DL})$$

$$H_{Actual} = 73.1 - (-1.5 + 3.2 + 4.1 + 13.8 + 0.1 + 0 + 1.24) = 52.2m$$

**Required  
Power  
[P<sub>Required</sub>]**

$$P_{Required} = \frac{Q_s' \times \rho \times H_{Act}}{102.04} + \sum Losses = \frac{(0.1045 + 0.00317) \times 973.6 \times 52.2}{102.04} + 20.7 = 74.3kW$$

**Pump  
Efficiency [η<sub>p</sub>]**

$$\eta_p = \frac{H_{Required}}{H_{Required} + H_{Losses}} = \frac{52.2}{52.2 + 20.9} \approx 71.5\%$$

## Nomenclature

$A_3$	Volute Throat Area [m <sup>2</sup> ]	$L_H$	Hydraulic Power [kW]
$A_L$	Leakage Area [m <sup>2</sup> ]	$m$	Mass flow rate [kg/s]
$b_1$	Impeller Passage Width at Inlet [m]	$N$	Rotational Speed [rpm]
$b_2$	Impeller Passage Width at Outlet [m]	$N_s$	Pump Specific Speed [rpm]
$b_3$	Volute Width [m]	$P_1$	Suction Flange Pressure [bara]
$C_L$	Leakage Loss Coefficient [-]	$P_2$	Discharge Flange Pressure [bara]
$C_m$	Disc Coefficient Friction [-]	$P_{\text{Required}}$	Power Required [kW]
$C_v$	Volute Flow Coefficient [-]	$Q_s$	Flow Rate [m <sup>3</sup> /s]
$d_s$	Diameter of Main Shaft End [m]	$Q_s'$	Total Flow Rate [m <sup>3</sup> /s]
$D_0$	Diameter of Impeller Eye [m]	$Q_N$	Maximum Flow Rate [m <sup>3</sup> /s]
$D_1$	Impeller Inner Diameter [m]	$R_1/R_2$	Radius Ratio [-]
$D_2$	Impeller Outer Diameter [m]	$Re$	Reynolds Number [-]
$D_2/D_1$	Stepanoff Coefficient	$s$	Axial gap [m]
$D_3$	Volute Mean Diameter [m]	$S_s$	Shaft permissible Shear Stress [psi]
$D_H$	Hub Diameter [m]	$t$	Blade Thickness [m]
$f$	Leakage Loss Coefficient	$t_1$	Thickness of Impeller Passage at Inlet [m]
$H$	Pump Head [m]	$t_2$	Thickness of Impeller Passage - Outlet [m]
$H_{\text{Circ}}$	Circulation Head Loss [m]	$U_2$	Impeller OD Tip Speed [m]
$H_{\text{In}}$	Incidence Head Loss [m]	$U_1$	Impeller ID Tip Speed [m]
$H_{\text{sf}}$	Surface Friction Head Loss [m]	$V_{\text{eye}}$	Velocity of Impeller Eye [m/s]
$H_{\text{vf}}$	Volute Friction Head Loss [m]	$V_{r1}$	Radial Velocity of Flow at Inlet [m/s]
$H_{\text{df}}$	Disc Friction head Loss [m]	$V_{r2}$	Radial Velocity of Flow at Outlet [m/s]
$H_{\text{dL}}$	Diffusion Head Loss [m]	$V_{\theta 1}$	Tangential Velocity of Flow at Inlet [m/s]
$H_L$	Leakage Head Loss [m]	$V_{\theta 2}$	Tangential Velocity of Flow - Outlet [m/s]
$H_{\text{RL}}$	Recirculation Head Loss [m]	$V_{\theta 1}'$	Actual Whirl Velocity Flow at Inlet [m/s]
$H_{\text{Actual}}$	Actual head Loss [m]	$V_{\theta 2}'$	Actual Whirl Velocity Flow at Outlet [m/s]
$H_R$	Hydraulic Radius [m]	$V_{s1}$	Slip Velocity at Inlet [m/s]
$k_s$	Disc Surface Roughness [m]	$V_{s2}$	Slip velocity at Outlet [m/s]
$K_u$	Stepanoff Coefficient	$W_1$	Relative Velocity at Inlet [m/s]
$k_{m1}$	Stepanoff Coefficient	$W_2$	Relative Velocity at Outlet [m/s]
$k_{m2}$	Stepanoff Coefficient	$Z$	Number of Impeller Vanes [-]
$L$	Shaft Power [kW]	$\beta_1$	Vane Angle at Inlet [degrees]
$L_{\text{AH}}$	Available Hydraulic Power [kW]	$\beta_2$	Vane Angle at Outlet [degrees]
		$\varepsilon_{\text{limit}1}$	Limiting Radius Ratio at Inlet [-]

$\varepsilon_{\text{limit2}}$	Limiting Radius Ratio at Outlet [-]
$\varepsilon_1$	Contraction Factor at Inlet [-]
$\varepsilon_2$	Contraction Factor at Outlet [-]
$\rho$	Liquid Density [kg/m <sup>3</sup> ]
$\eta_p$	Pump Efficiency [%]
$\eta_v$	Volumetric Efficiency [%]
$\sigma_{s1}$	Slip Value at Inlet [-]
$\sigma_{s2}$	Slip Value at Outlet [-]
$\mu$	Liquid Viscosity [kg/m.s]
$\omega$	Angular Velocity [m/s]

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## About the Author



Vijay Sarathy holds a Master's Degree in Chemical Engineering from Birla Institute of Technology & Science (BITS), Pilani, India and is a Chartered Engineer from the Institution of Chemical Engineers, UK. His expertise over 10 years of professional experience covers Front End Engineering, Process Dynamic Simulation and Subsea/Onshore pipeline flow assurance in the Oil and Gas industry. Vijay has worked as an Upstream Process Engineer with major conglomerates of General Electric, ENI Saipem and Shell.